Sequential versus independent commitment*
– An indirect evolutionary analysis of bargaining rules –

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June, 1997
revised version: January, 1998

Abstract

Rational bargaining behavior depends crucially on the rules of bargaining, especially on whether parties decide sequentially or independently. Whereas in ultimatum bargaining the proposer can exploit the responder, independent commitments result in more balanced payoffs. To limit the scope of possible bargaining results we try to rule out certain bargaining rules. In our indirect evolutionary analysis we first determine the solution for all possible rule constellations and then derive the evolutionary stable rules of bargaining. It is shown that ultimatum bargaining requires considerable, but non-maximal uncertainty about the size of the pie, i.e. the monetary amount to be distributed.

*The author gratefully acknowledges helpful comments by Sandra Güth, Eric van Damme and Rolf Tschernig. Support from the Deutsche Forschungsgemeinschaft (SFB 373, Quantifikation und Simulation ökonomischer Prozesse) is gratefully acknowledged.

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1. Introduction

According to its non-cooperative approach bargaining theory should model the bargaining process as a strategic game and then determine rational bargaining behavior by selecting among the equilibria – in case there exists a multiplicity of equilibria (Nash, 1953). Famous examples are Nash's own model of simultaneous commitments, which usually has many equilibria of which one can select one by applying the Nash (1953)-bargaining solution, or the alternating offer model (Ståhl, 1982, Krelle, 1976, and – more elegantly –Rubinstein, 1982) with a unique subgame perfect equilibrium.

The wide variety of bargaining rules is troublesome since different bargaining rules usually imply widely different outcomes. In ultimatum bargaining, where one party can confront the other with a take it or leave it – offer, the proposer, for instance, acquires essentially all the reward whereas simultaneous commitments, as modelled and analyzed by Nash (1953), result in more balanced payoffs. To limit this troublesome variety of bargaining outcomes we apply the indirect evolutionary approach allowing to derive the evolutionarily stable bargaining rules.

To demonstrate how this can be done we rely on a simple bargaining situation in which parties can try to preempt the other or to restrain themselves. Preemption, however, means to risk conflict since one has to commit one's own demand before the random choice of the **pie**, i.e. of the total monetary reward. In the tradition of the **indirect evolutionary approach** (Güth and Yaari, 1992) we first determine the solution of all possible constellations of such timing dispositions and then derive the evolutionarily disposition – preemption aiming at ultimatum bargaining or retention aiming at independent commitments?

The literature on **endogenous timing of decisions** (see, for instance, van Damme and Hurkens, 1996, and the references there, especially Spencer and Brander, 1992, and – with a different subgame structure – Sadanand and Sadanand, 1996) can be seen as an alternative method of limiting the wide scope of possible bargaining rules and outcomes. Here the selection of bargaining rules is not governed by evolutionary pressure, but part of the strategic calculus of the interacting parties themselves. One therefore needs an overall game model capturing the rule choices as well as the behavioral choices for all possible rule constellations. An advantage of the indirect evolutionary approach is that one does not have to specify such an overall game model. Actually none of the parties involved needs to know how evolutionary forces select among bargaining rules. Furthermore, the indirect evolutionary approach usually implies different results even when the structural relationships are known (see Dufwenberg and Güth, 1997).

In section 2 we introduce our simple bargaining environment. Section 3 is devoted to solving all possible bargaining games, and section 4 to the discussion of the evolutionarily stable constellations. In section 5 we summarize our results and indicate some possible lines of generalizing them.

2. The bargaining situation

Let us denote the two bargaining parties as player 1 and 2. Each party $i = 1, 2$ can develop as two types – genotypes in evolutionary terminology. A U-type tries to confront the other party with an ultimatum offer whereas an I-type prefers to wait. Accordingly there are three possibilities of behavioral dispositions m_i with

(1) $\quad m_i \in \{U, I\}$ for $i = 1, 2$.

In case of $(m_1, m_2) = (I, I)$ both parties $i = 1, 2$ determine simultaneously their demand $d_i \geq 0$ after the random choice of the pie c, i.e. the total monetary reward which can be distributed among the two parties. If the available total reward c satisfies $d_1 + d_2 \leq c$, each party $i = 1, 2$ receives d_i whereas 0-payoffs result in case of **conflict**, i.e. for $d_1 + d_2 > c$.

If $(m_1, m_2) = (U, U)$, i.e. if both parties try to preempt the other, the demands d_i must be chosen before the realization of the stochastic variable c. Of course, no party then can actually preempt the other since both parties determine their demand early and thus independently.

In case of $(m_1, m_2) = (U, I)$ or (I, U) the U-type party can confront the other party with a take it or leave it – proposal d_U which the other I-party can only accept or reject (the latter implying 0-payoffs for both). Here it is important that the U-type must commit itself before c is randomly selected, i.e. a high demand may imply conflict.

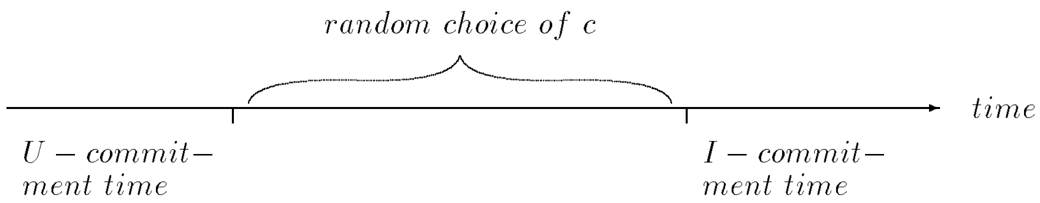

Figure 1: The timing structure of a U- and I- disposition as related to the random choice of c.

In figure 1 the timing structure is graphically visualized. Bargaining parties only play sequentially in case of asymmetric timing dispositions with the U-type preempting the I-type, but risking conflict since the U-type must commit itself before c is randomly selected.

Whenever the timing dispositions are the same, parties must commit themselves independently. In case of U-types this is done before the random choice of c what will result in conflict when demands are positive and the pie c is too small. If both parties are of the I-types, conflict can be avoided since the pie c is commonly known when parties commit themselves independently.

We first consider the simple case where $c \in [0, 1]$ is uniformly distributed on $[0, 1]$ what is commonly known. In general, let both parties be risk neutral, i.e. the (positive) marginal utilities for money are constant. Thus for both parties, the cardinal utility is measured by the respective expected monetary payoff.

Only in case of $(m_1, m_2) = (I, I)$ we will confront an essential non-uniqueness of equilibrium behavior. By imposing symmetry invariance and efficiency we will then select a unique bargaining outcome similar to Nash (1950 and 1953). For the other (m_1, m_2)-constellations no selection among equilibria is needed.

3. Solution behavior

Let us start with $(m_1, m_2) = (I, I)$ when both parties $i = 1, 2$ determine their demand $d_i \geq 0$ after the random choice of c, i.e. when choosing d_i party $i = 1, 2$ knows c. The are many equilibrium demand vectors (d_1, d_2), namely all vectors (d_1, d_2) with $d_1 + d_2 = c$ and $d_1, d_2 \geq 0$, but also $d_1 = c = d_2$. Clearly, efficiency requires

$$(2) \qquad d_1 + d_2 = c \text{ for all } c \in [0, 1] \, .$$

Efficiency and symmetry (invariance) together therefore imply demands $d_i^* (c)$ such that

$$(3) \qquad d_i^* (c) = c/2 \text{ for all } c \in [0, 1] \, .$$

The obvious advantage of $(m_1, m_2) = (I, I)$ is that conflict is always avoided since the expected payoffs of both parties $i = 1, 2$ are

$$(4) \qquad u_i (I, I) = \int_0^1 \frac{c}{2} dc = \frac{1}{4} \text{ for } i = 1, 2.$$

Since the solutions for the (U, U)- and the (U, I)-constellations will be shown to resemble the well-known duopoly market equilibria of Cournot (1838) and von Stackelberg (1951), respectively, one may wonder about the duopoly market interpretation of the (I, I)-constellation. Due to the efficiency requirement the (I, I)-result resembles the optimal exploitation of customers on the market, i.e. the two sellers jointly engage in perfect price discrimination and – by the symmetry requirement – share their revenues equally. For the normalized linear demand function with a prohibitive price and a satiation level of 1, perfect price discrimination of all customers yields revenues of 1/2; thus each seller receives 1/4 as the two bargaining parties in case of the (I, I)-constellation.

In case of $(m_1, m_2) = (U, U)$ demands d_i with $i = 1, 2$ are chosen before the random choice of c. For a given demand vector (d_1, d_2) party i's expected payoff u_i is thus

(5) $\quad u_i(d_1, d_2) = \int\limits_{d_1+d_2}^{1} d_i dc = d_i(1 - d_1 - d_2)$ for $i = 1, 2.$

From the necessary and sufficient condition for a maximum of $u_i(d_1, d_2)$ with respect to d_i for $i = 1, 2$ one obtains

(6) $\quad 1 - 2d_1^* - d_2^* = 0 = 1 - 2d_2^* - d_1^*$

and thus the well-known duopoly-solution (Cournot, 1838)

(7) $\quad d_i^* = 1/3$ for $i = 1, 2$

resulting in

(8) $\quad u_i(d_1^*, d_2^*) = 1/9$ for $i = 1, 2.$

We now turn to the final case of one U- and one I-type where – without loss of generality – we can analyze only $(m_1, m_2) = (U, I)$. Party 1, the U-type, must commit itself to a demand d_1 before c is randomly determined. But when deciding whether to accept or reject d_1, party 2 – the I-type – knows the realization of c, i.e. party 2 responds to the ultimatum d_1 after the random choice of c. Thus party 2 only accepts d_1 if $c \geq d_1$ and rejects d_1 otherwise. Consequently, party 1's expected payoff is

(9) $\quad u_1(d_1) = \int\limits_{d_1}^{1} d_1 dc = d_1(1 - d_1).$

Thus the optimal ultimatum proposal is the demand

(10) $\quad d_1^* = 1/2.$

The resulting expected payoffs are

(11) $\quad u_1(d_1^*) = 1/4$ and

$$(12) \qquad u_2^* = \int_{1/2}^{1} (c - 1/2)\, dc = 1/8,$$

similar to the well-known sequential duopoly-solution (von Stackelberg, 1951).

With these results we have determined the expected payoffs u_i for both parties i and all possible (m_1, m_2)-constellations with $m_i \in \{U, I\}$ for $i = 1, 2$. In the terminology of the indirect evolutionary approach this means that we have completed its first step what allows us to define an evolutionary game with reproductive successes u_i and types $m_i \in \{U, I\}$ for the two interacting parties $i = 1, 2$.

4. Evolutionary analysis

The evolutionary game with reproductive success u_i and mutants $m_i \in \{U, I\}$ for $i = 1, 2$ is determined by the results of section 3 where we have derived the expected payoffs u_i for all constellations (m_1, m_2) with $m_i \in \{U, I\}$ for $i = 1, 2$.

m_1 \ m_2	U	I
U	$\frac{1}{9}, \frac{1}{9}$	$\frac{1}{4}, \frac{1}{8}$
I	$\frac{1}{8}, \frac{1}{4}$	$\frac{1}{4}, \frac{1}{4}$

Table 1: The evolutionary game in bimatrix form

Table 1 represents the evolutionary game in bimatrix form with entries (u_1, u_2). Whereas $m_i = I$ achieves a higher expected payoff than $m_i = U$ when the other party is of the U-type, the same payoff is achieved when the other party is of the I-type. We thus have proved

Proposition 1: For the bargaining situation, described in section 1, there exists only one evolutionarily stable bargaining disposition, namely the I-type.

Proof: In the table $m_i = I$ weakly dominates $m_i = U$ what proves that the bimatrix game has a unique evolutionarily stable m_i-type in the sense of an evolutionarily stable strategy, namely $m_i = I$ for $i = 1, 2$ due to

$$1/4 = u_1(I, I) = u_1(U, I) = \frac{1}{4}$$

and

$$1/8 = u_1(I, U) > u_1(U, U) = 1/9.$$

■

In the long run both parties will thus be of the I-type, i.e. the bargaining rule will be the one of simultaneous commitments after the random choice of c. Neither early simultaneous commitments nor the ultimatum rule are evolutionarily stable since the final population must be I-monomorphic: Finally, since both parties wait for the random choice of c before they commit themselves and thereby avoid conflict and always distribute the whole "pie" c, the only efficient constellation of timing dispositions is evolutionarily stable.

In view of endogenous timing Table 1 would not describe an evolutionary game, but the truncation of an overall game where later subgames are substituted by the solution payoffs as determined in section 3 (see, for instance, Table 2 of Spencer and Brander, 1992, p. 1611). Such an overall game would have to assume that parties $i = 1, 2$ strategically choose their timing disposition $m_i \in \{U, I\}$ before the U-commitment time in the figure above. In general, it will matter whether such strategic choices of m_i are made independently or sequentially and what the parties know about each other, but since $m_i = I$ weakly dominates $m_i = U$ for the case at hand such details do not matter. Thus our analysis can also be viewed as a study of endogenous timing in bargaining situations (see the more or less related results of Spencer and Brander, 1992, as well as Sadanand and Sadanand, 1996).

5. Reducing the degree of uncertainty

Since in Table 1 one earns the same expected profit as an $m_i = U$ or an $m_i = I$-type when the other party is of the I-type, Proposition 1 could describe a rather special result. On the other hand a similar analysis for general densities on $[0,1]$ is rather difficult (see Appendix). When analysing how the degree of uncertainty influences the result, we therefore rely on a special class of densities, namely the uniform densities on $\left[\frac{1}{2}-\delta, \frac{1}{2}+\delta\right]$ with $0 \leq \delta \leq \frac{1}{2}$. Our previous result refers to the special case of $\delta = \frac{1}{2}$. Compared to this boundary case positive parameters $\delta < \frac{1}{2}$ express more certain expectations, i.e. less uncertainty regarding the size of the pie.

Proceeding as for $\delta = \frac{1}{2}$ one obtains the following results:

In case of $(m_1, m_2) = (I, I)$ the result does not change since for all parameters δ with $0 \leq \delta \leq \frac{1}{2}$ the expected pie is $\frac{1}{2}$. Thus one obtains $d_i^*(c) = c/2$ and $u_i(I, I) = \frac{1}{4}$ for $i = 1, 2$.

In case of $(m_1, m_2) = (U, U)$ one obtains

$$(13) \qquad d_i^* = \begin{cases} \frac{1}{6} + \frac{\delta}{3} \text{ for } \frac{1}{2} \geq \delta \geq \frac{1}{10} \\ \frac{1}{4} - \frac{\delta}{2} \text{ for } \frac{1}{10} \geq \delta \geq 0 \end{cases}$$

and

$$(14) \qquad u_i(d_1^*, d_2^*) = \begin{cases} \frac{\left(\frac{1}{6}+\frac{\delta}{3}\right)^2}{2\delta} \text{ for } \frac{1}{2} \geq \delta \geq \frac{1}{10} \\ \frac{1}{4} - \frac{\delta}{2} \text{ for } \frac{1}{10} \geq \delta \geq 0 \end{cases}$$

for $i = 1, 2$.

Finally, the result for $(m_1, m_2) = (U, I)$ is described by $d_2^*(c) = c - d_1$ if $c \geq d_1$ as well as by

$$(15) \qquad d_1^* = \begin{cases} \frac{1}{4} + \frac{\delta}{2} \text{ for } \frac{1}{2} \geq \delta \geq \frac{1}{6} \\ \frac{1}{2} - \delta \text{ for } \frac{1}{6} \geq \delta \geq 0 \end{cases}$$

yielding

$$(16) \qquad u_1\left(d_1^*\right) = \begin{cases} \frac{\left(\frac{1}{4}+\frac{\delta}{2}\right)^2}{2\delta} \text{ for } \frac{1}{2} \geq \delta \geq \frac{1}{6} \\ \frac{1}{2} - \delta \text{ for } \frac{1}{6} \geq \delta \geq 0 \end{cases}$$

and

$$(17) \qquad u_2^* = \begin{cases} \frac{\left(\frac{1}{4}+\frac{\delta}{2}\right)^2}{4\delta} \text{ for } \frac{1}{2} \geq \delta \geq \frac{1}{6} \\ \delta \text{ for } \frac{1}{6} \geq \delta \geq 0 \end{cases} .$$

One easily can check our former results by setting $\delta = \frac{1}{2}$. The limiting case of no uncertainty at all, i.e. $\delta = 0$, yields the evolutionary game of Table 2 with the only equilibrium (U, U) in strictly dominant strategies.

m_1 \ m_2	U	I
U	$\frac{1}{4}, \frac{1}{4}$	$\frac{1}{2}, 0$
I	$0, \frac{1}{2}$	$\frac{1}{4}, \frac{1}{4}$

Table 2: The special case of certainty ($\delta = 0$)

For parameters δ with $\frac{1}{6} \leq \delta \leq \frac{1}{2}$ the corresponding evolutionary game is described by

m_1 \ m_2	U	I
U	$\frac{\left(\frac{1}{6}+\frac{\delta}{3}\right)^2}{2\delta}, \frac{\left(\frac{1}{6}+\frac{\delta}{3}\right)^2}{2\delta}$	$\frac{\frac{1}{4}\left(\frac{1}{2}+\delta\right)^2}{2\delta}, \frac{\frac{1}{2}\left(\frac{1}{4}+\frac{\delta}{2}\right)^2}{2\delta}$
I	$\frac{\frac{1}{2}\left(\frac{1}{4}+\frac{\delta}{2}\right)^2}{2\delta}, \frac{\frac{1}{4}\left(\frac{1}{2}+\delta\right)^2}{2\delta}$	$\frac{1}{4}, \frac{1}{4}$

Table 3: The case of uncertainty with $\frac{1}{6} \leq \delta \leq \frac{1}{2}$

Notice that for $\frac{1}{6} \leq \delta < \frac{1}{2}$ one always has

$$(18) \qquad \frac{1}{2}\left(\frac{1}{4}+\frac{\delta}{2}\right)^2/2\delta > \left(\frac{1}{6}+\frac{\delta}{3}\right)^2/2\delta$$

as well as

$$(19) \qquad \frac{1}{4}\left(\frac{1}{2}+\delta\right)^2/2\delta > \frac{1}{4}$$

what proves that (U, I) and (I, U) are strict equilibria.

For $0 < \delta \leq 1/10$ the evolutionary game is given by Table 4.

m_1 \ m_2	U	I
U	$\frac{1}{4}-\frac{\delta}{2}, \frac{1}{4}-\frac{\delta}{2}$	$\frac{1}{2}-\delta, \delta$
I	$\delta, \frac{1}{2}-\delta$	$\frac{1}{4}, \frac{1}{4}$

Table 4: The case of uncertainty with $0 < \delta \leq 1/10$

Since U strictly dominates I due to $\delta \leq 1/10$, only $(m_1^*, m_2^*) = (U, U)$ is evolutionarily stable.

For the remaining case $1/10 \leq \delta \leq 1/6$ the evolutionary game is described by Table 5. Here the result depends on

m_1 \ m_2	U	I
U	$\frac{\left(\frac{1}{6}+\frac{\delta}{3}\right)^2}{2\delta}, \frac{\left(\frac{1}{6}+\frac{\delta}{3}\right)^2}{2\delta}$	$\frac{1}{2}-\delta, \delta$
I	$\delta, \frac{1}{2}-\delta$	$\frac{1}{4}, \frac{1}{4}$

Table 5: The case of uncertainty with $1/10 \leq \delta \leq 1/6$

whether

$$(20) \qquad \left(\tfrac{1}{6}+\tfrac{\delta}{3}\right)^2/2\delta \geq \delta$$

holds or not. Now inequality (20) is equivalent to

$$(20') \qquad \left(\tfrac{1}{6}>\right)\tfrac{1+3\sqrt{2}}{34} \geq \delta.$$

Thus Table 5 has a unique solution in dominating strategies, namely $(m_1^*, m_2^*) = (U, U)$, for

$$(21) \qquad \tfrac{1+3\sqrt{2}}{34} \geq \delta \geq 0,$$

and two strict equilibria, namely $(m_1^*, m_2^*) = (U, I)$ and $(m_1^*, m_2^*) = (I, U)$, for

$$(22) \qquad \tfrac{1}{6} > \delta > \tfrac{1+3\sqrt{2}}{34}.$$

In case of two strict equilibria (and an additional equilibrium in completely mixed strategies) the interpretation and implications of evolutionary stability could depend on whether one assumes a single population-model (both players 1 and 2 are drawn from the same large population and randomly matched) or relies on two populations (each player $i = 1, 2$ is drawn from his own large population and randomly matched with a partner, randomly drawn from the other population). Here we do not want to exclude the evolutionary stability of the two asymmetric strict equilibria (U, I) and (I, U) by imposing the one population-model. In evolutionary terminology this means that there exist two species, one representing player 1 and the other representing player 2. Given this interpretation our previous results imply

Proposition 2: For the bargaining situation, described in section 1, the evolutionarily stable timing dispositions $m_i^* \in \{U, I\}$ for $i = 1, 2$ depend on the degree of pie uncertainty as follows:

(i) In case of small uncertainty $\left(0 \leq \delta \leq \frac{1+3\sqrt{2}}{34}\right)$ the only stable timing dispositions are those of the monomorphic constellation $(m_1^*, m_2^*) = (U, U)$.

(ii) In case of greater uncertainty $\left(\frac{1+3\sqrt{2}}{34} < \delta < \frac{1}{2}\right)$ only the two bimorphic timing dispositions $(m_1^*, m_2^*) = (U, I)$ and $(m_1^*, m_2^*) = (I, U)$ are stable, i.e. the rules of bargaining are those of ultimatum bargaining.

(iii) Only if uncertainty is maximal $\left(\delta = \frac{1}{2}\right)$ the unique stable constellation is the monomorphic constellation $(m_1^*, m_2^*) = (I, I)$.

Proof: According to Table 2 for $\delta = 0$ as well as Tables 4 and 5 for $\delta > 0$ the evolutionary game has a unique equilibrium in dominating strategies $m_i^* = U$ if $\delta \leq \frac{1+3\sqrt{2}}{34}$ what proves (i). For part (ii) the evolutionary game has two strict equilibria due to (18) and (19), respectively (22), namely $(m_1^*, m_2^*) = (U, I)$ and $(m_1^*, m_2^*) = (I, U)$, and a completely mixed strategy equilibrium which, by definition, is non-strict. Clearly both, $m_i = U$ and $m_i = I$, are alternative best replies against this mixed equilibrium strategy. If now the (mixed equilibrium strategy-)population of one player is gradually invaded by either the U- or the I-mutant, this would bring about an evolutionary advantage of the I-, respectively the U-type of the other population, as compared to the mixed equilibrium strategy. Thus according to the two population-interpretation the mixed equilibrium strategy is evolutionarily unstable whereas there exist no alternative best replies for the two strict equilibria (U, I) and (I, U). This proves that only $(m_1^*, m_2^*) = (U, I)$ and $(m_1^*, m_2^*) = (I, U)$ are evolutionarily stable. Part (iii) simply restates Proposition 1. ■

In view of Proposition 2 ultimatum bargaining (see Güth, 1976, and Güth, Schmittberger, and Schwarze, 1982, for the first experimental analysis) requires essential, but non-maximal pie uncertainty. Here one, however, should keep in mind that our analysis has been restricted to the class of uniform distributions. There is no loss of generality in assuming $c \in [0, 1]$ since the pie must be non-negative and since one can set the maximal possible pie for all possible distributions with finite carrier equal to 1. But one can induce even more pie uncertainty than in case of the uniform density with $\delta = \frac{1}{2}$ by allowing for non-uniform densities. Clearly, more pie uncertainty in this sense will render general retention, i.e. $(m_1^*, m_2^*) = (I, I)$, a generic result and not simply one of a border case $\left(\delta = \frac{1}{2}\right)$.

6. Final remarks

One potential of the indirect evolutionary approach is that it allows to derive the evolutionarily stable rules of the game which are usually exogenously imposed. We have accomplished the first step of indirect evolution by requiring rationality in the form of (subgame perfect or efficient and symmetry invariant) equilibrium behavior.

For the second step – the determination of the evolutionarily stable m_i-types – we have imposed no essential restriction. Most reasonable learning, cultural, or genetical evolution dynamics and all static concepts of evolutionary stability (see Hammerstein and Selten, 1994, as well as Weibull, 1995, for surveys) imply that the constellations, as described by Propositions 1 and 2, are evolutionarily stable.

One may add that in view of the (indirect) evolutionary perspective a multiplicity of stable timing configurations is not troublesome: What finally results may depend on initial conditions. Since in view of endogenous timing such an ambiguity is more troublesome, one may want to resolve players' strategic uncertainty by applying the theory of equilibrium selection (e.g. Harsanyi and Selten, 1988). In case of part (ii) of Proposition 2 symmetry implies the completely mixed strategy equilibrium (of the truncation), i.e. all four m_1, m_2-constellations with $m_i \in \{U, I\}$ for $i = 1, 2$ would result with positive probability.

The study demonstrates how one easily can derive the dynamic process of bargaining which usually has to be exogenously imposed and for which there exist many possible specifications with widely varying implications for the bargaining outcome. Earlier applications of the indirect evolutionary approach have usually concentrated on the evolution of preferences, e.g. Güth and Yaari (1992), Güth and Kliemt (1994), Bester and Güth (forthcoming), or on the evolution of beliefs (Güth, 1995) and not on the extensive form of the game.

To illustrate how the dynamic process of bargaining can be derived, we have relied on simplifying assumptions like uniform densities governing the random choice of c. Except for computational difficulties one can easily generalize our analysis. The optimality conditions for more general distributions are derived and discussed in the Appendix which, in the cases (U,U) and (I,I), do not allow an easy computation of the profit expectations and thus of the evolutionary bimatrix game.

One may worry, more specifically, about the result in Proposition 1 since according to Table 1 the timing disposition I is only weakly dominant. Would the I-type become strictly dominant if one allows for larger variances than the one implied by $\delta = \frac{1}{2}$? There are various possibilities to explore this question, e.g. by assuming uniform densities on the intervals $\left[0, \frac{1}{2} - \delta\right]$ and $\left[\frac{1}{2} + \delta, 1\right]$ for $0 < \delta \leq \frac{1}{2}$. Another possibility (see figure 2) are linear densities $f(c)$ on $[0,1]$ of the form $f(c) = 1 + \delta - 4\delta c$ for $0 \leq c \leq \frac{1}{2}$ and $f(c) = 1 - \delta + 4\delta c$ for $\frac{1}{2} \leq c \leq 1$ where $-1 \leq \delta \leq 1$. Both versions contain the situation in Proposition 1 as the special case $\delta = 0$. And the variance of c will be larger than the one assumed by Proposition 1 if δ is positive. We leave it to the reader to answer the question above for such specific densities.

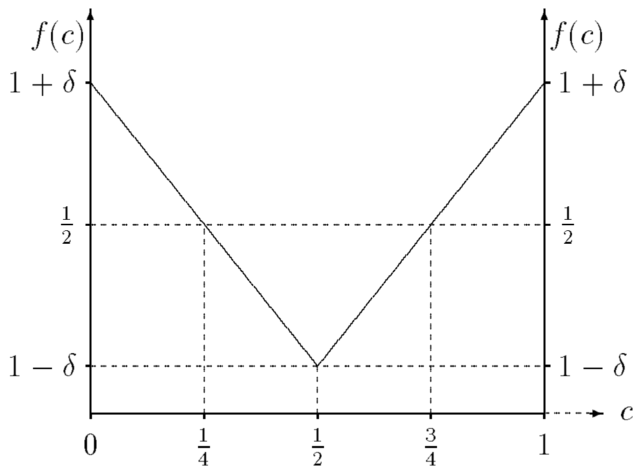

Figure 2: Linear densities $f(c) = 1 + \delta - 4\delta c$ for $0 \leq c \leq \frac{1}{2}$ and $f(c) = 1 - \delta + 4\delta \left(c - \frac{1}{2}\right)$ allowing for larger variances in case of $0 < \delta \leq 1$ than the uniform density on $[0,1]$.

A further assumption is that timing dispositions are commonly known (see Güth and Kliemt, 1994, for an indirect evolutionary analysis with incomplete information and van Damme, 1997, for an endogeneous timing analysis, based on our model, which assumes that timing and demand decisions are made simultaneously).

Here our aim has been to demonstrate the following message in the simplest way: Dear Reader, if you are troubled by the multiplicity of possible bargaining rules and its resulting multiplicity of possible bargaining outcomes, you can derive the evolutionarily stable bargaining rules and thus limit the scope of possible bargaining outcomes by applying the indirect evolutionary approach!

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Appendix: Optimality conditions for densities $f(c)$ on $[0,1]$ with $f(c) = \mathbb{F}'(c) = F'(c)$

For (I,I) one has $d_i^*(c) = c/2$ and therefore (by integration by parts)

$$\begin{aligned} U_i^* &= \int_0^1 \frac{c}{2} f(c)\, dc = \frac{1}{2}\left(cF(c) \mid_0^1 - \mathbb{F}(c) \mid_0^1 \right) \\ &= \frac{1}{2}\left(1 - \mathbb{F}(1) + \mathbb{F}(0)\right). \end{aligned}$$

For (U,U) the payoff function of party 1 is

$$U_1(d_1, d_2) = d_1 \int_{d_1+d_2}^{1} f(c)\, dc$$

so that

$$\frac{\partial}{\partial d_1} U_1(d_1, d_2) = \int_{d_1+d_2}^{1} f(c)\, dc - d_1 f(d_1 + d_2) = 0.$$

If

$$\frac{\partial^2}{\partial d_1^2} U_1(d_1, d_2) = -2f(d_1 + d_2) - d_1 f'(d_1 + d_2) < 0,$$

one obtains by symmetry

$$\int_{2d}^{1} f(c)\, dc = df(2d)$$

$$1 = F(2d^*) + d^* f(2d^*) \text{ for } d^* = d_1^* = d_2^*$$

as an implicit equation for the solution $d^* = d_1^* = d_2^*$ in case of (U, U).

Similarly, for (I, U) one has

$$U_1(d_1) = d_1 \int_{d_1}^{c} f(c)\, dc$$

so that

$$U_1'(d_1) = \int_{d_1}^{1} f(c)\, dc - d_1 f(d_1) = 0.$$

Provided that

$$U_1''(d_1) = -2f(d_1) - d_1 f'(d_1) < 0$$

one thus has

$$1 = F(d_1^*) + d_1^* f(d_1^*)$$

as an implicit formula for the solution d_1^* in case of (U, I).

Since in the two latter cases (U, U) and (U, I) the optimal decision d_i^* are only defined by implicit formulae, an explicit equation for $U_i(d_1^*, d_2^*)$ in case of (U, U) and of $U_1(d_1^*)$ as well as

$$U_2^* = \int_{d_1^*}^{1} (c - d_1^*) f(c)\, dc$$

in case of (U, I) is not readily available.